

Sylvan Learning Center Math Research

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The National Council of Teachers of Mathematics (NCTM) released their document Principles and Standards for School Mathematics in 2000. This is an update of their previous documents Curriculum and Evaluation Standards for School Mathematics (1989), Professional Standards for Teaching Mathematics (1991), and Assessment Standards for School Mathematics (1995). It is based on educational research, professional expertise and amassed classroom experience. The writing group included teachers, teacher educators, administrators, researchers and mathematicians.

The Principles and Standards for School Mathematics include six principles that define elements of high quality mathematics programs. The ten standards define the knowledge, understanding and skills that all students from grades pre-kindergarten through grade twelve should acquire. The standards are divided into five content standards and five process standards. Sylvan's math programs, Sylvan Math Essentials (K-8) and Sylvan's advanced math programs, are directly aligned with the NCTM's findings.

The following summary of the NCTM's findings, as well as other scientific research, follows the structure of the Principles and Standards for School Mathematics and explains how Sylvan Learning Center is directly aligned with those findings.

Principles for School Mathematics

The Equity Principle

- Excellence in mathematics education requires equity— high expectations and strong support for all students.

Kilpatrick, J., Swafford, J., & Bradford, F. (Eds.). (2001). *Adding it all up: Helping children learn mathematics*. Washington, DC: National Academies Press.

- "The overriding premise of our work is that throughout the grades from pre-k through eight all students can and should be mathematically proficient." (p.409)

Grouws, D.A., & Cebulla, K.J. (2000). *Improving student achievement in mathematics* (Report No. SE 064 318). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education. (ERIC Document Reproduction Service No. EDO-SE-00-10)

- Students' opportunity to learn bears directly on their mathematics achievement.

The Sylvan Principles and Methods for Equity

A traditional widespread belief was that some students will always have trouble learning mathematics. This has been disproved by research. All students can learn mathematics. Sylvan mathematics programs use a variety of instructional methods, which address different learning styles and increase a student's ability to learn via a variety of methods. Sylvan provides the

individual help that students need to become proficient math students and to develop a love of learning. The stated goal of Sylvan Math Essentials is to “facilitate the development of mathematical competency in all students.”

The Curriculum Principle

- A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

U.S. Department of Education. (1997). Mathematics equals opportunity [White Paper prepared for U.S. Secretary of Education]. Washington, DC: Author.

- Students who take rigorous mathematics and science courses are much more likely to go to college than those who do not.
- Algebra is the “gateway” to advanced mathematics and science in high school, yet most students do not take it in middle school.
- Mathematics achievement depends on the courses a student takes, not the type of school the student attends.

Masini, B. & Taylor, J. (2000). New evidence links curricula and instruction to mathematics achievement (Report No. SE 065-062). Oak Brook, IL: North Central Regional Educational Lab. (ED 455-110)

- In analyzing results from the Third International Mathematics and Science Study (TIMSS), the percent of topics covered prior to eighth grade was positively related to mathematics achievement. Likewise, the percent of topics new at the eighth grade was negatively related to mathematics achievement.

- Covering algebra in middle school is strongly associated with achievement growth throughout high school, regardless of math skills before high school.
- Curricula should provide students opportunities to learn key content at an early age.

The Sylvan Principles and Methods for Curriculum

Sylvan’s mathematics curriculum is a coherent curriculum that flows from kindergarten through trigonometry and pre-calculus. Connections are made in the curriculum between related mathematics skills. Fractions and decimals are used throughout instruction and reinforced in the algebra curriculum. The NCTM content and process standards are included in the Sylvan mathematics curriculum and are introduced at the developmentally appropriate age and grade level.

The Teaching Principle

- Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Masini, B. & Taylor, J. (2000). New evidence links curricula and instruction to mathematics achievement (Report No. SE 065-062). Oak Brook, IL: North Central Regional Educational Lab. (ED 455-110)

- Teachers need to teach by asking probing questions to develop students’ conceptual understanding.

The Sylvan Principles and Methods for Teaching

Sylvan mathematics teachers are trained and qualified. They use a collection of instructional methods to meet different learning styles. Sylvan mathematics teachers combine diagnostic assessments, ongoing assessments and a strong curriculum to develop customized programs for students. Since Sylvan teachers are giving the students individual attention they are able to challenge and encourage students fully.

The Learning Principle

- Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Kilpatrick, J., Swafford, J., & Bradford, F. (Eds.).(2001). *Adding it all up: Helping children learn mathematics*. Washington, DC: National Academies Press.

- All students should be mathematically proficient. Mathematical proficiency consists of five components.
- Conceptual understanding — comprehension of mathematical concepts, operations, and relations
- Procedural fluency — skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence — ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning — capacity for logical thought, reflection, explanation, and justification
- Productive disposition — habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Masini, B.& Taylor, J. (2000).*New evidence links curricula and instruction to mathematics achievement* (Report No. SE 065-062).Oak Brook, IL: North Central Regional Educational Lab.(ED 455-110)

- Curricula must support instructional standards that emphasize a deep conceptual understanding over simply skill building.
- Students need to write about their ideas, justify their solutions and explore strategies for problem solving.

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- Studies have consistently shown that an emphasis on teaching for meaning has a positive effect on initial understanding, retention of the concept and application of the knowledge in new situations.

The Sylvan Principles and Methods for Learning

All Sylvan Math Essentials lessons start by accessing prior knowledge. Students learn new skills by building on the understanding of related math skills. Sylvan students then demonstrate skill mastery prior to advancing to the next skills. Sylvan Math Essentials students develop both a procedural fluency and a conceptual understanding of computation. Both Sylvan Math Essentials and the advanced mathematics course lessons have opportunities for the students to work concretely, using manipulatives, understanding the concept. Then the student moves to procedural fluency. The students then apply the skill in problem solving activities. All of the Sylvan mathematics courses have a writing component to encourage students to synthesize, reflect and communicate their ideas mathematically.

The Assessment Principle

- Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

The Sylvan Principles and Methods for Assessment

All of the Sylvan mathematics programs use diagnostic and ongoing assessments to develop customized content for students.

The Technology Principle

- Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

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- The use of calculators can increase student achievement in mathematics and positively affect student attitudes.
- One valuable use for calculators is in exploration and discovery of new topics.
- Students should be allowed to use their calculators in achievement tests. It is the way they often do mathematics and will continue to do so when they are not in a school setting.

The Sylvan Principles and Methods for Technology

Sylvan mathematics courses incorporate the use of both calculators and computer software to reinforce and augment learning. Students must become efficient and accurate with basic computation. As students are developing basic computational algorithms, they need to learn paper and pencil strategies, mental math strategies, and estimation strategies. While students are learning basic computation, calculators should not be used. It is also important to teach students when the use of a calculator is most efficient. Sylvan Math Essentials students, after developing a proficiency at computation, use calculators to explore new concepts and when a calculator is more efficient. Graphing calculators are an integral part of the Sylvan advanced math courses.

Standards for School Mathematics

Instructional programs from pre-kindergarten through grade 12 should enable all students to do the following:

The Problem Solving Standard

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

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- Students learn concepts and skills while working on problem solving exercises.
- It is not necessary for the skills to be developed first and then later move on to applying the skill.

The Sylvan Methods for Teaching Problem Solving

Problem solving is an integral part of all Sylvan mathematics courses. For Sylvan Math Essentials, problem solving is a part of every lesson; part of the process of learning a new skill is then to apply it in problem solving exercises. In addition, Sylvan Math Essentials has specific problem solving objectives. Students are taught to choose and use an appropriate problem solving strategy. This is carried through the advanced mathematics courses.

The Reasoning and Proof Standard

- recognize reasoning and proofs as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

The Sylvan Methods for Teaching Reasoning and Proof

Sylvan mathematics students are taught to look for patterns, make conjectures and test their theories.

The Communication Standard

- organize and consolidate mathematical thinking through communication;
- communicate mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

The Sylvan Methods for Communication

A daily part of all the Sylvan mathematics courses is for students to write about the skills they have been learning. It is an opportunity for students to reflect on what they have been learning and communicate it precisely using mathematical terms. Throughout the lessons, especially during the Guided Practice and error analysis steps, teachers are asking questions, analyzing the students' thinking and encouraging them to communicate effectively.

The Connections Standard

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

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- Focusing instruction on the meaningful development of important mathematical ideas increases the level of student learning.
- Instruction should emphasize how the skill is connected to other mathematical ideas.

The Sylvan Methods for Teaching Connections

All of the Sylvan Math Essentials lessons start by linking the new skill to prior knowledge. Connections are built in all the Sylvan mathematics courses within math, between different disciplines and to everyday life.

The Representation Standard

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

The Sylvan Methods for Teaching Representation

Students routinely use pictures or pictorial representations as they move from initial concept development to abstract problem solving. Students also regularly use charts, graphs and tables to organize data and solve problems. The following are the five NCTM content standards. All of these are exemplified in the Sylvan Math Essentials and the Sylvan advanced math courses' Intended Learning Outcomes (ILOs).

The Number and Operations Standard

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimates.

The Algebra Standard

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

The Geometry Standard

- analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

The Measurement and Data Analysis Standard

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

The Data Analysis and Probability Standard

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Below are two additional topics that are important in mathematics teaching methods.

Discovery and Practice

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- Balance is important between students' discovery of new concepts and practice of skills and methods that they have been taught.
- When students invent and discover mathematical concepts they have a stronger conceptual understanding of connections between mathematical ideas.

The Sylvan Methods for Discovery and Practice

Sylvan's mathematics courses develop concepts starting with a concrete approach then move to pictorial and abstract. The teaching is a balance between students exploring new concepts with concrete materials and students developing procedural fluency with algorithms.

The Use of Concrete Materials

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- Regular use of manipulatives helps students develop useful meanings for mathematical ideas.
- Students' use of manipulatives should not be limited to repeating demonstrations. Exploring the concept with concrete materials in meaningful ways is essential.
- Long term use of manipulatives improves students' understanding of mathematical concepts and attitudes to mathematics.

Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology*, 23(4), 270.

- Students understand mathematics and have greater gains in mathematics when manipulatives are used with third graders.

The Sylvan Methods for Using Concrete Materials

Manipulatives are a vital part of Sylvan Math Essentials. The manipulatives are used when a new skill is introduced. Teachers ask questions to have the students explore new concepts and explain skills with the manipulatives. The use of manipulatives continues into Sylvan's advanced math courses with tools such as Algeblocks.

Conclusion

All of Sylvan's mathematics courses are based on scientific research and aligned with the NCTM's principles and standards. Sylvan believes that all students can be proficient in mathematics. With a coherent curriculum, qualified teachers, diagnostic assessments and balanced and proven teaching methods, Sylvan Learning Centers develop customized programs for students and help them reach high expectations.

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